Quantitative Measures of Change based on Feature Organization: 
Eigenvalues and Eigenvectors

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I. Introduction

Man made objects are organized; even their placement tends to be regular. For example, buildings exhibit geometric forms, road widths vary slowly, airplanes are parked parallel to one another, and a full parking lot has a regular structure. This 3D world structure manifests as regular 2D image feature organizations. We explore the role of this emergent structure or organization among the 2D image features in change detection. Specifically, we address the problem of differentiating between no development, onset of construction, and full development. Prior work in change detection include [4, 5, 6, 7].

Fig. 1 shows a fully developed site, an undeveloped forest site, and a site under construction. Note the differences in the organization among the edge features. The edges in the fully developed site exhibit more parallelism, continuity, closure, and perpendicularity than those in the undeveloped site. Also, the edge segments in the fully developed site are generally larger than those in the undeveloped site. We quantify these qualitative differences using measures which capture both the statistics of the individual edge features and the relationships among them. The underlying theory is based on the spectra (eigenvalues and eigenvectors) of graphs. The relationships among the image edge features are represented as a Relation Graph. The eigenvalues and the eigenvectors of the adjacency matrix of this graph provide us with measures which capture the global relationship among the edge features.

The use of eigenvalues and eigenvectors in computer vision is not new. The concept of eigenvalue has been

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II. The Relation Graph

The global relationship among the image features can be very effectively captured in the form of a graph whose nodes represent the image features and whose links denote compatibility between the features. Two image features are said to be compatible if they exhibit pairwise organization, e.g. the two structures are of the same type, similar size, and have similar orientation (generalized parallelism). We call this graph the Relation Graph.

We consider constant curvature edge segments as the primitive image features. The weighted links between the features capture the regular Gestalt inspired relationships of proximity, parallelism, closure, strands, perpendicularity, and continuity. A closure relationship (N-ary) exists among a set of edge segments if they enclose a region. A strand relationship (N-ary) exists among a set of edge segments if they form a contiguous chain of segments without completely enclosing a region, e.g. any three sides of a rectangle. We use the bottom-up algorithm described in [10] to find the Gestalt relationships and quantify their significance using the expressions in Fig. 3.

We denote any two edge segments by $e_i$ and $e_j$ and their lengths by $l_i$ and $l_j$, respectively. Fig. 2 illustrates the concepts used in the expressions of Fig. 3. Each significance measure incorporates the significance of the constituent edge segments denoted by $\text{Sig}(e_i)$ which is computed using the segment length, $l_i$, and the least root mean squared fit error, $\text{Err}_i$, (in units of pixels), of a constant curvature segment. Each term in the expressions of Fig. 3 capture an essential component of the corresponding relationship definition. The overall significance of a relation is determined by the minimum value.

The significance of a parallelism relation ($\text{Sig}_{par}$) between two edge segments is determined by the difference in length between the constituting edge segments, the length of the symmetry axis, $l_{sym}$, the average width, $\mu_w$, and width standard deviation, $\sigma_w$, of the parallel strip, and the minimum fit error, $\text{Err}_{sym}$, of a constant curvature segment to the symmetry axis. The symmetry axis is computed as described in [11].

The significance of a closure relationship ($\text{Sig}_{clo}$) or a strand relationship ($\text{Sig}_{str}$) is dependent on the the amount of edge support we have for the perimeter of the enclosed region which is determined by the length of the perimeter of the region ($l_p$) enclosed by the edge segments and the total gap ($l_g$) between the end points of the consecutive edge segments forming the boundary (as shown in Fig. 2(d)). The closure relationship also incorporates a convexity term which involves the area of the convex hull of the region, $a_{hull}$, and the actual area enclosed, $a_{actual}$.

The perpendicularity ($\text{Sig}_{per}$) and continuity relation ($\text{Sig}_{con}$) are captured using the minimum gap between them, $d_{gap}$, and the angles between the segments as shown in Figs. 2(e) and (f).

Next, we combine the quantified Gestalt relations to generate the link weights of the Relation graph. The link weight $w(e_i, e_j)$ between two nodes is given by:

$$
\text{Sig}(e_i) = \frac{l_i}{l_i + \text{Err}_i}
$$

$$
\text{Sig}_{proxi}(e_i, e_j) = 1 - \min(1, \frac{l_{\min}}{\min(l_i, l_j)})
$$

$$
\text{Sig}_{par}(e_i, e_j) = \min \left( \frac{\text{Sig}(e_i), \text{Sig}(e_j), \mu_w \pm \sigma_w}{l_{sym}}, \frac{l_{sym} + \text{Err}_{sym}}{l_{sym} + \text{Err}_{sym} + l_{sym} + l_i + l_j} \right)
$$

$$
\text{Sig}_{clo}(e_i, \ldots, e_k) = \min \left( \frac{\text{Sig}(e_i), \ldots, \text{Sig}(e_j)}{l_{p}}, \frac{l_p}{l_i + l_j} \right)
$$

$$
\text{Sig}_{str}(e_i, \ldots, e_k) = \min \left( \frac{\text{Sig}(e_i), \ldots, \text{Sig}(e_j)}{l_{p}}, \frac{l_p}{l_i + l_j} \right)
$$

$$
\text{Sig}_{per}(e_i, e_j) = \min \left( \frac{\text{Sig}(e_i), \text{Sig}(e_j), \frac{l_i + l_j}{\mu_w + \sigma_w}}{\sin^2(\theta_i)} \right)
$$

$$
\text{Sig}_{con}(e_i, e_j) = \min \left( \frac{\text{Sig}(e_i), \ldots, \text{Sig}(e_j)}{l_i + l_j + d_{gap} \cos^2(\theta_i)} \right)
$$
\[ w(c_i, c_j) = \text{Sig}_{\text{prox}}(c_i, c_j) \cdot |\text{Sig}_{\text{pare}}(c_i, c_j) + \text{Sig}_{\text{geo}}(c_i, c_j) + \text{Sig}_{\text{attr}}(c_i, c_j) + \text{Sig}_{\text{geo}}(c_i, c_j) + \text{Sig}_{\text{con}}(c_i, c_j)|^2 \]

Note that the relations involved in Eq. (1) are binary, except for the closure and strand relationships. However, the link weights, \( w(c_i, c_j) \), are defined pairwise. To facilitate this for \( N \)-ary relations, we use the following convention: \( \text{Sig}_{\text{geo}}(c_{i_1}, c_{i_2}) = \text{Sig}_{\text{geo}}(c_{i_1}, \cdots, c_{i_k}) \) for every \( \{i, j\} \in \{k_1, \cdots, k_2\} \).

That is, all pairs of the edges involved in an \( N \)-ary (\( N > 2 \)) relation are assigned a significance value equal to that for the whole set.

Another important aspect of the combination rule in Eq. (1) is that the proximity factor multiplies the sum of the other factors. Thus, if the proximity factor, \( \text{Sig}_{\text{prox}} \), between two features is zero, the final weight is zero irrespective of the values of the other factors. The multiplication effects ANDing of the proximity term with the other terms which are ORed together using addition. This has the effect of giving priority to local relations. It is also possible to weight each of the individual terms in the expression according to the importance of a particular relation in a given domain. However, we have found the expression in its present form to perform satisfactorily.

III. Theory: Graph Spectra

Eigenvalues and eigenvectors of the relation graph provide exciting possibilities as a basis for measures of change. The concept of graph eigenvalues is motivated as follows [8]. Let us consider the task of finding node clusters in a weighted graph \( G \) of \( n \) nodes. Given a node-cluster, we can motivate a cluster cohesiveness measure in the following way.

Let us represent a node cluster using a column vector, \( x \), whose \( k \)-th entry captures the participation of node \( k \) in that cluster; we allow for a graded membership of a node in a cluster. If a node does not participate in a cluster, the corresponding entry is zero. We impose the restriction that the norm of this weight vector \( x \) be one or \( x^T x = 1 \). Then, based on the link weights of the graph, \( w_{ij} \), we can define the following measure for the cohesiveness of the node cluster:

\[ \sum_{i=1}^{n} \sum_{j=1}^{n} w_{ij} x_i x_j = x^T A x \]  

where \( A \) is the weighted adjacency matrix (which is Hermitian \( A = A^T \)). Note that since the entries in \( x \) corresponding to the non-participating nodes in a cluster are zero, essentially only a submatrix of \( A \) will be involved in the above equation. The more well-connected the cluster nodes are, the larger is the cluster cohesiveness.

A maximally cohesive cluster \( x \) can be found by maximizing the above expression. The Rayleigh-Ritz theorem [9, page 176] states that the maximum value of the above expression will be \( \lambda_{\text{max}} \), the maximum eigenvalue \(^1\) of \( A \), and the corresponding eigenvector will be the optimal \( x \). In fact, the theorem also implies that the minimum value will be \( \lambda_{\text{min}} \), the minimum eigenvalue of \( A \).

In general, we will have \( n \) eigenvalues, which is the total number of nodes in the graph. The (non-decreasing) ordered set of eigenvalues \( \{\lambda_1, \cdots, \lambda_n\} \) is referred to as the spectrum of the graph. An interpretation for all the eigenvalues and eigenvectors of \( A \) can be given using the Courant-Fischer theorem [9, page 179], which is a generalization of the Rayleigh-Ritz theorem. Let \( \{x_{\lambda_1}, \cdots, x_{\lambda_{n-k+1}}\} \) denote the set of eigenvectors corresponding to the \( k \)-largest eigenvalues, \( (\lambda_{n-k}, \cdots, \lambda_{n-k+1}) \), respectively. Then,

\[ \lambda_{n-k} = \max_{x\perp\{x_{\lambda_1}, \cdots, x_{\lambda_{n-k+1}}\}} x^T A x \]

Thus, the \( k \)-th largest eigenvalue, \( \lambda_{n-k} \), is the maximum value of the cluster cohesiveness measure for all node weight assignment vectors \( x \), that are orthogonal to the eigenvectors with the \( (k-1) \)-largest eigenvalues.

For example, the components of the eigenvector with the second largest eigenvalue give a cluster weight assignment which is orthogonal to the cluster weight assignment (eigenvector) with the largest eigenvalue. In practical terms, we can use this orthogonality constraint to define clusters with the least amount of overlap between them.

Thus, we see that the spectrum of a graph provides us with a natural clustering of its nodes. The components of an eigenvector denote the node participation in the associated cluster, while the corresponding eigenvalue denotes the cohesiveness of the cluster. If the nodes participating in a cluster are all connected to each other with weight one (a perfectly coherent group), then its eigenvalue is the maximum possible, the number of nodes in the group minus one. The eigenvalue decreases as the interconnections get sparser. Thus, the larger the eigenvalue, the greater the coherence and the size of its associated cluster.

\(^1\) For any \( N \times N \) matrix \( A \), eigenvalues are constants \( \lambda \) such that \( A x = \lambda x \) for a column vector \( x \), the corresponding eigenvector.
IV. Measures for monitoring change

With scene change, specifically construction change, not only will the spectrum of the corresponding relation graph evolve but also the individual feature attributes will change. For example, we expect the average lengths of the constant curvature segments to change as construction progresses. We use these feature attributes along with the eigenvalues and eigenvectors to formulate measures to monitor change due to construction.

First, the maximum eigenvalue, $\lambda_{\text{max}}$, is an indicator of the largest amount of structure in an image. This should be large for a fully developed site.

Second, we consider the size weighted average eigenvalue, $\lambda_{\text{size}}$, of the eigenvectors. Let the eigenvalues of the eigencenters be denoted by $\{\lambda_i^c, i = 0, \ldots, N_c\}$, where $N_c$ is the total number of clusters. Also, let the total number of edge segments participating in the $i^{th}$ eigencenter be $n_i$. Then $\lambda_{\text{size}} = \frac{\sum_{i=1}^{N_c} \lambda_i^c n_i}{n}$. This measure will be large for images having large eigencenters with large eigenvalues, i.e., for images with large groups of edges, each exhibiting high organization.

Third, we compute a measure of the average length given by $I_{\text{tot}} = \sum_{i=1}^{N_c} \frac{1}{n_i} \left(\sum_{j=1}^{n_i} l_{ij}\right)$ where $l_{ij}$ is the length of the $j^{th}$ edge segment in the $i^{th}$ eigencenter, normalized by the square root of the image area to account for differences in image resolution. It is the total length of the edge segments participating in all the eigencenters and gives an indication of the fraction of image exhibiting organization.

Fourth, we consider a distance measure $\text{Dist}(\mathcal{G}_t, \mathcal{G}_{t-1})$ between the Relation graph at the present moment, $\mathcal{G}_t$, and the Relation graph at a previous time, $\mathcal{G}_{t-1}$. To compare the spectra from two images we build on the formulation in [12]. Let the number of nodes in graphs $\mathcal{G}_t$ and $\mathcal{G}_{t-1}$ be $n_1$ and $n_2$, respectively, with $\{\lambda_1, \ldots, \lambda_{n_1}\}$ and $\{\lambda_1^2, \ldots, \lambda_{n_2}^2\}$ as their spectra, respectively. Without loss of generality we assume $n_1 > n_2$.

To compare the eigenvalues from the two graphs of different sizes, we insert zeroes into the smaller spectrum until the two spectra are of the same length. This is equivalent to adding isolated nodes in the smaller graph. Let the new, padded eigenvalue sequence be $\{\mu_1, \ldots, \mu_{n_1}\}$. Then, a measure of the distance between two spectra can be given by $\text{Dist}(\mathcal{G}_t, \mathcal{G}_{t-1}) = \ldots$ 

Note that [12] considered graphs with the same number of nodes. We generalize the measure to consider graphs with different numbers of nodes.

An example of the eigencenters is shown in Fig. 4, which displays the 6 best eigencenters detected in the image shown in Fig. 1(a). The darkness of an edge segment in a cluster is proportional to the participation of the segment. The participation of an edge in a cluster is proportional to the corresponding component in the eigenvector. Note that the eigencenters correspond to salient structures such as buildings and roads.

Figure 4: Eigencenters detected in Fig. 1(a). The eigencenters detected with eigenvalues (a) 6.87, 3.42, 2.98, (b) 5.22, (c) 2.44, and (d) 2.22.

In general, the components of an eigenvector will not be all positive. Eigenvectors with negative components are not meaningful in the above physical interpretation of a graph spectrum since the membership of a node in a cluster cannot be negatively valued. Thus, in this paper, we consider only positive eigenvectors. An eigenvector $x_i$ is said to be positive if all the components of $x_i$ or $-x_i$ are positive and its corresponding eigenvalue is positive. (Note that if $x$ is an eigenvector then so is $-x$.) We refer to the components of a positive eigenvector which account for most (95%) of its total squared value as the dominant components. We call the features corresponding to these dominant components the active features participating in the eigenvector. We refer to this set of active features corresponding to the dominant components of a positive eigenvector as an Eigencenter. The relative weight of each active feature in a cluster is determined by the modulus of the corresponding component value in the eigenvector. This weight is a measure of how much a feature "belongs" to an eigencenter; it measures the participation of the part in the whole. The eigencenters thus defined will be disjoint since they are derived from the non-zero components of the positive eigenvectors which are, as described in a previous section, orthogonal to each other.
\[ \left( \sum_{i=1}^{n} (\lambda_i - \mu))^2 \right)^{1/2} \]. It can be shown [12] that this is also a measure of the distance between the two graphs. Note that we need not bring the nodes in the two graphs into correspondence to compute this measure. The measure is 0 for isomorphic graphs.

V. Results

In this section we explore the use of the proposed four measures in differentiating among: an undeveloped site, a site with the onset of construction, and a fully developed site. As a test bed we used the publicly available RADIUS imagery. We considered 14 aerial images of a fully developed region with a building and 11 images of an undeveloped region under different viewpoints, resolutions, time of day, and cloud conditions. The image set includes images taken in the early morning with long shadows, images under cloud cover, and images with different resolution and viewpoints. We also considered a set of 11 images of a site under construction over 42 days. The set covers the ground clearing stage to the laying of the foundation walls. This set of images covers the preliminary construction stage.

The graph spectrum has to be robust with respect to random perturbations of the relation graph links, but, it should be sensitive to systematic change in the underlying graph structure. In other words, the eigenvalues need to exhibit invariance (at least quasi-invariance) with respect to imaging conditions such as viewpoint, resolution, time of day, and weather but should change with change in the underlying scene. In fact, stability is guaranteed under random perturbations by Weyl's theorem [9, page 367] which states that the variation of the eigenvalues of a perturbed matrix is bounded by the maximum and the minimum eigenvalues of the perturbing matrix. We have studied the effect of varying the link weights in a systematic manner to model construction activity and have found the graph spectra to be sensitive to changes in the statistical nature of the distribution of the link weights. This is particularly important since with progress in construction activity we expect not only the emergence of new relationships between image features but we also expect the relationships to grow stronger.

For each of the images in our test sets we computed the edges, the Gestalt graphs, the relation graph, and its associated eigenvalues. The plot in Fig. 5 shows the variation of the positive eigenvalues for the fully developed site and the undeveloped site. We show error bars at one standard deviation on either side of the mean value. The x-axis of the plot corresponds to the ranking of the eigenvalues; the largest eigenvalue being considered first. Note that the variation is modest given the range of imaging conditions we consider. Also, note that the graph spectra of the fully developed site and the undeveloped sites are clearly separated (more on this later). For eigenvalues greater than one, the average ratio of the standard deviation to the mean for each eigenvalue is 0.11. For eigenvalues less than one, the variation ratio is larger at 0.40. However in practice, eigenvalues less than one does not represent meaningful underlying clusters.

The box and whisker plots in Fig. 6 shows the variation of the four measures of change. The top and bottom of each box in the plots are located at the sample 25th and 75th percentiles. Thus, each box contains roughly half of the sample values. The horizontal lines drawn within the boxes mark the 50th percentiles, or medians. The plus signs (+) mark the means of the sample values. The vertical lines, or whiskers, extend from the boxes as far as the sample values extend, to a distance of at most 1.5 interquartile ranges. Fig. 6(d) shows the spread of the \textbf{Dist}(\mathcal{G}, \mathcal{G}_{-1}) measure as computed from an undeveloped site. Thus, the first box from the left is for the intraclass distances for an undeveloped site and the next two boxes are for interclass distances between a site with onset of construction and an undeveloped site, and between a fully developed site and an undeveloped site, respectively.

We can observe that no development can be clearly discriminated from full development. Also the values of the measures for the site under construction lie between those for an undeveloped forest site and a fully developed site. The mean values of the measures of a site under construction are separated from those of an undeveloped and fully developed site.
Figure 6: Variation of the four measures within the three classes: no development, onset of construction, and full development denoted by 0, 1, and 2, respectively, on the horizontal axes. Variation of $\lambda_{\text{max}}$ is shown in (a), of $\lambda_{\text{size}}$ in (b), of $l_{\text{tot}}$ in (c), and $\text{Dist}(G_t, G_{t-1})$ in (d).

Execution Times: The algorithm is implemented in C and has been tested on a Sun Sparc-20 workstation. The average (total) execution time over all the images in this paper was 97 seconds including file I/O and user interaction times. The average (total) CPU time was 71 seconds. Of this, an average of 8 seconds was spent on edge detection and segmentation, and 16 seconds of CPU time was spent on preattentive organization. The average CPU time spent computing the eigenvalues and eigenvectors was 35 seconds.

VI. Conclusion

In this paper we proposed four measures of image organizational change which can be used to monitor construction activity. The measures are based on the thesis that the progress of construction will see a change in the individual image feature attributes as well as an evolution in the relationships among these features. This change in the relationship is captured by the eigenvalues and eigenvectors of the relation graph embodying the organization among the image features. We demonstrate the ability of the measures to differentiate between no development, the onset of construction, and full development, on the available real test image set. In the future, we plan to concentrate on strategies to distinguish between the incremental local changes in construction activity such as those observed at access road construction, ground clearing, excavation, foundation wall erection, or roofing stage.

VII. References


